Vendor Selection by Application of Revised Weighting Method and Fuzzy Multicriteria Linear Programming

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Vendor selection in supply chain is a multicriteria problem that involves a number of quantitative and qualitative factors. This work deals with a concrete problem of flour purchase by a company that manufactures bakery products. The criteria for vendor selection and quantities supplied by individual vendors are: purchase costs, product quality and reliability of vendor. The problem of vendor selection is solved by a model that combines revised weighting method and fuzzy linear programming. The study points to the advantages of using the combination of the two methods in comparison to the separate use of one of them only.

Keywords: vendor selection, fuzzy linear programming, revised weighting method

1. Introduction

The problem of vendor selection and determination of material quantities supplied is the key element in the purchasing process in manufacturing which is one of the most important activities in supply chain. If all the selected vendors are able to meet the buyer's requirements completely, then the selection process becomes easier and is based only on the selection of the most suitable vendor in terms of purchasing costs, product quality, and vendor reliability. Nevertheless, practice shows that it is not good to rely on one vendor only. Therefore the management of the purchasing company generally enters into contracts with several vendors. Their number usually ranges from two to five for each sort of material. Also, there are cases when no vendor can meet the buyer's demand, or will not do it in order to protect his own business interests.

In principle there are two kinds of supplier (vendor) selection problem: The first is supplier selection when there is no constraint or in other words all suppliers can satisfy the buyer's requirements of demand, quality, delivery etc. In this kind of supplier selection the management needs to make only one decision - which supplier

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is the best one. The other type of supplier selection problem is when there are some limitations on suppliers' capacity, quality and so on. In other words, no supplier can satisfy the buyer's total requirements and the buyer needs to purchase some of the needed material from one supplier and some from another to compensate for the shortage of capacity or low quality of the first supplier. Consequently, the firm must decide which vendors it should contract with and it must determine the appropriate order quantity for each vendor selected.

In this paper we will discuss the second kind of supplier selection problem, but we will also provide a model which allows making both decisions simultaneously. The model combines two methods used in operational researches. The first of them, revised weighting method is used to determine the coefficient weights of complex criteria functions (cost, quality and reliability). Coefficients determined in this way present the coefficients of the objective functions in the fuzzy multi-criteria programming model providing the final selection and the quantity supplied from a particular vendor. The constraints in the multiple objective programming model are the total demand and the limitations of supplier capacities.

High competition forces companies to produce faster, at less cost, and better than their competitors, which cannot be done unless they select the best vendors. The increasing importance of vendors makes companies consider a number of criteria in vendor selection. A list of criteria for vendor selection can be seen in the classic study by Weber et al.(1991), or for example in the paper from Lin and Chen (2004) who generate a Generic Configuration Hierarchy (GCH) of supplier attributes that could be used as the basis for supplier selection criteria in any industry. They list no fewer than 138 such attributes.

The literature dealing with vendor selection uses various methods. Among the numerous studies dealing with this topic we will mention some more important ones. The AHP method was used for vendor selection in the following works: Narasimhan 1983, Nydick-Hill 1992 and Barbarosoglu-Yazac 1997. For vendor evaluation Weber-Ellram 1992, Weber-Desai 1996, and Weber et al. 1998 use the DEA method. The fuzzy AHP method for vendor evaluation is used in the studies by Haq-Kannan (2006) and Chan-Kumar (2007). For vendor evaluation and determination of supply quotas the AHP is used in combination of some methods of mathematical programming. Thus for instance Ghodsypour and O'Brien (1998) use the AHP method in combination with linear programming. Ge Wang et al. (2004) use the AHP and goal programming. Kumar et al. (2008) use the AHP method and fuzzy linear programming, while Kumar et al. (2004, 2005) use only fuzzy goal programming for that purpose.

Obviously, vendor selection is an important issue dealt with by numerous researchers. Great efforts are made to define appropriate models for vendor selection and determination of supply quotas from the selected vendors and to apply the adequate methods to solve such models.

The aims of this work are the following: (1) to point on the concrete example that vendor selection is a multicriteria problem, (2) to propose criteria for vendor selection, (3) to propose the model for vendor selection and determination of supply quotas by using the revised weighting method and fuzzy linear programming, and (4) to point to the advantages of the proposed model in comparison to the usual methods of vendor selection. The concrete example will be the problem of flour vendor selection by a bakery.

The rest of the paper is organized as follows: We will first present the methodology of vendor selection and determination of supply quotas by use of revised weighting method and fuzzy linear programming. Then the proposed methodology will be tested on the concrete example of vendor selection by a bakery. Finally, we will carry out sensitivity analysis of the obtained solutions. In the conclusion we will point to the advantages of using the proposed methodology in comparison to the use of revised weighting method, or fuzzy linear programming method only.

2. Methodology of vendor selection and determination of supplied quantity

For vendor selection and determination of supplied quantity we will use the revised weighting method and fuzzy linear programming (FLP). The revised weighting method is used to determine the coefficient weights of complex criteria functions. The coefficients obtained in this way are used as criteria functions coefficients in the LP fuzzy model. The main steps in the proposed model are:

1. Determining criteria for vendor selection,
2. Applying revised weighting method to determine the variable's coefficients in criteria functions,
3. Building and solving the FLP model to determine supply quotas from selected vendors,
4. Sensitivity analysis of the obtained solution.

2.1. Determining criteria for vendor selection

The first step in the proposed methodology is selection of criteria for vendor selection. Numerous criteria are stated in literature and their selection depends on the concrete problem (Weber et al. 1991). The most important criteria may certainly be: the total purchasing costs in a particular period, product quality offered by particular vendors, and vendor reliability. Each of these criteria is expressed through a number of sub-criteria, which can further be expressed through a number of sub-sub-criteria, etc. This reveals the hierarchical structure of criteria for vendor selection, which directs us to apply the revised weighting method to solve this problem (Koski-Silvennoinen 1987).

2.2. The revised weighting method

We will give a brief outline of the basic propositions of this multicriteria method used in a large number of factual cases.

The main idea of the weight coefficient method as presented by Gass and Satty (1955) and Zadeh (1963) is to relate each criteria function with the weight coefficient and to maximize/minimize the weighted sum of the objectives. In that way the model containing several criteria functions is transformed into the model with one criteria function. It is assumed that the weight coefficients w_j are real numbers so that $w_j \geq 0$ for all $j = 1, \ldots, k.$ It is also assumed that the weights are normalized, so that $\sum_{j=1}^{k} w_j = 1.$ Analytically presented, the multicriteria model is modified into a monocriterion model and is called the weight model:

$$\max/\min \sum_{j=1}^{k} w_j f_j(\underline{x}) = \sum_{j=1}^{k} \sum_{i=1}^{n} w_j c_{ij} x_i \tag{1}$$

s.t

$$\underline{x} \in X, \tag{2}$$

where $w_j \geq 0$ for all $j = 1, \ldots, k,$ $\sum_{j=1}^{k} w_j = 1.$ To make the weight coefficients w_j express the relative importance of criteria functions f_j we propose linear transformation of criteria functions coefficients. To allow addition of weighted criteria functions we have to transform all of them either into functions that have to be maximized or into functions to be minimized. Linear transformation of criteria functions coefficients that have to be maximized is performed in the following way:

$$c_{ij}^{'} = c_{ij} / c_j^{*}, \tag{3}$$

where $c_j^{*} = \max_i c_{ij}.$ Obviously $0 \leq c_{ij}^{'} \leq 1.$

The criteria functions that have to be minimized will be transformed into functions to be maximized by taking reciprocal values of coefficients $1/c_{ij}.$ Then

$$c_{ij}^{'} = \frac{1/c_{ij}}{\max_i (1/c_{ij})} = \frac{\min_i}{c_{ij}} = \frac{c_j^{\min}}{c_{ij}}. \tag{4}$$

Now we will normalize the coefficients $c_{ij}^{'}$ into dimensionless space $[0,1]$:

$$c''_{ij} = \frac{c'_{ij}}{\sum_{i=1}^{n} c'_{ij}}, \quad j = 1,\ldots,k. \tag{5}$$

The above transformations allow us to obtain the weighted sum of criteria functions in which the weights reflect the relative importance of criteria functions.

It is to be noted that in the revised method of weight coefficients all theoretical results valid in the basic weight coefficient method remain valid. We will here present the three basic theorems in the light of the revised weight coefficient method.

Theorem 1: Solution of the weight model (1-2) is weakly Pareto optimal.

Proof: The proof will be shown for the case of maximization. Let $\underline{x}^* \in X$ be the solution of the weight model. Let us assume that the solution is not weakly Pareto optimal. In such a case there is solution $\underline{x} \in X$ so that $f_j(\underline{x}) > f(\underline{x}^*)$ for all $j = 1,\ldots k,$ because we have $w_j > 0$ for at least one $j.$ Consequently, $\sum_{j=1}^{k} w_j f_j(\underline{x}) > \sum_{j=1}^{k} w_j f_j(\underline{x}^*).$ This contradicts the assumption that $\underline{x}^*$ is the solution of the weight model. Therefore, $\underline{x}^*$ is the weakly Pareto optimal solution.

Theorem 2: The solution of the weight model (1-2) is Pareto optimal if all the weight coefficients are positive, i.e. $w_j > 0$ for all $j = 1,\ldots k.$

Proof: Let $\underline{x}^* \in X$ be the solution of the weight model with positive weight coefficients. Let us assume that this solution is not Pareto optimal. This means that there is another solution $\underline{x} \in X$ so that $f_j(\underline{x}) \geq f(\underline{x}^*)$ for all $j = 1,\ldots k,$ and $f_j(\underline{x}) > f(\underline{x}^*)$ for at least one $j.$ As $w_j > 0$ for all $j = 1,\ldots k,$ we get $\sum_{j=1}^{k} w_j f_j(\underline{x}) > \sum_{j=1}^{k} w_j f_j(\underline{x}^*).$ This contradicts the assumption that $\underline{x}^*$ is the solution of the weight model and therefore has to be Pareto optimal (Miettinen 1999).

Theorem 3: The unique solution of the model (1-2) is Pareto optimal.

Proof: Let $\underline{x}^* \in X$ be the unique solution of the weight model. Let us assume that it is not Pareto optimal. In that case there is solution $\hat{\underline{x}} \in X$ so that $f_j(\hat{\underline{x}}) \geq f(\underline{x}^*)$ for all $j = 1,\ldots k,$ and $f_j(\hat{\underline{x}}) > f(\underline{x}^*)$ for at least one $j.$ Because all the coefficients w_j

are non-negative, we have $\sum_{j=1}^{k} w_j f_j(\hat{\underline{x}}) \geq \sum_{j=1}^{k} w_j f_j(\underline{x}^*)$. On the other hand, the uniqueness of $\underline{x}^*$ means that $\sum_{j=1}^{k} w_j f_j(\underline{x}^*) > \sum_{j=1}^{k} w_j f_j(\hat{\underline{x}})$ for all $\hat{\underline{x}} \in X$. These two inequations are contradictory, therefore $\underline{x}^*$ has to be Pareto optimal.

In this paper we use the revised weight coefficients method to reduce the number of complex criteria functions. This idea originates from Koski and Silvennoinen (1987). According to it, the normalized original criteria functions are divided into groups so that the linear combination of criteria functions in each group forms a new criteria function while the linear combination of new criteria functions form a further criteria function, etc. In this way we obtain a model with a reduced number of criteria functions. According to this each Pareto optimal solution of the new model is also Pareto optimal solution of the original model, but the reverse result is not generally true.

The weight coefficients method has some shortcomings so that it is not the most appropriate one to create a set of Pareto optimal solutions. The shortcomings are: (1) varying weight coefficients do not guarantee that we will determine all Pareto optimal solutions, and (2) the determined Pareto optimal solutions are those that are situated in the extreme points of the convex polyhedron but not those that connect the two extreme points. To determine the set of compromise solutions and the preferred solution, we will here use the fuzzy linear programming method.

2.3. *Fuzzy linear programming (FLP)*

The general multi-criteria programming model to solve the problem of determining the supply quotas by selected vendors can be presented as:

Find the vector $\underline{x}$ which minimizes criteria functions f_r and maximizes criteria functions f_s with

$$f_r = \sum_{i=1}^{n} c_{ri} x_i, \qquad r = 1, 2, \ldots, p \tag{6}$$

$$f_s = \sum_{i=1}^{n} c_{si} x_i, \qquad s = p+1, p+2, \ldots, q \tag{7}$$

s.t.

$$\underline{x} \in X_d, \tag{8}$$

where $X_d = \left\{ \begin{array}{l} \underline{x} \mid g_l(\underline{x}) = \sum_{i=1}^{n} a_{il} x_i \geq b_l,\, g_p(\underline{x}) = \sum_{i=1}^{n} x_i = D,\, 0 \leq x_i \leq u_i,\, l = 1,\ldots,m, \\ i = 1,2,\ldots,n \end{array} \right\}$, and

x_i is the quantity ordered from the vendor *i*, *D* is the total demand in the given period, u_i is the upper limit of order from the vendor *i*, c_{ri} are the coefficients with variables in criteria functions that are to be maximized, such as: Total Value of Purchasing (TVP), product quality, vendor reliability, etc., while c_{si} are coefficients with variables in criteria functions that have to be minimized, such as total purchasing costs, etc., and a_{il} are coefficients in constraints, which can for instance be vendor flexibility in terms of delivery quotas, subjective rating of the vendor, etc., while b_l represents the lower limit of constraint $g_l(\underline{x})$.

Zimmermann 1978 solved the problem (6 – 8) by fuzzy linear programming approach. He formulated the fuzzy linear program determining for each criteria function f_j its maximal value f_j^+ and its minimal value f_j^-, solving:

$$f_r^+ = \max f_r, \quad \underline{x} \in X_a, \quad f_r^- = \min f_r, \quad \underline{x} \in X_d \tag{9}$$

$$f_s^+ = \max f_s, \quad \underline{x} \in X_d, \quad f_s^- = \min f_s, \quad \underline{x} \in X_a \tag{10}$$

f_r^-, f_s^+ will be obtained by solving the multicriteria model as linear programming model separately minimizing or maximizing single criteria functions. $\underline{x} \in X_d$ means that solutions belongs to feasible set X_d, while X_a is a set of all optimal solutions obtained by solving single criteria functions.

As for each criteria function f_j its value is changed linearly from f_j^- to f_j^+, that value can be observed as a fuzzy number with linear membership function $\mu_{f_j}(\underline{x})$.

Consequently, the MCLP model (6 – 8) with fuzzy goals and fuzzy constraints can be presented as:

$$\tilde{\tilde{f}}_r = \sum_{i=1}^{n} c_{ri} x_i \leq\approx f_r^0, \qquad r = 1,2,\ldots,p \tag{11}$$

$$\tilde{\tilde{f}}_s = \sum_{i=1}^{n} c_{si} x_i \geq\approx f_s^0, \qquad s = p+1, p+2,\ldots,q \tag{12}$$

s.t.

$$\tilde{\tilde{g}}_l(\underline{x}) = \sum_{i=1}^{n} a_{il} x_i \geq\approx b_l, \quad l = 1,\ldots,m \tag{13}$$

$$\tilde{\tilde{g}}_p(\underline{x}) = \sum_{i=1}^{n} x_i =\approx D, \ 0 \le x_i \le u_i, \quad i = 1,\ldots,n. \tag{14}$$

In this model the sign $\approx$ indicates fuzzy environment. The symbol $\le\approx$ denotes the fuzzy version $\le$, and is interpreted as “essentially smaller than or equal to”, the symbol $\ge\approx$ is interpreted as “essentially greater than or equal to”, while the symbol $=\approx$ is interpreted as “essentially equal to”. f_r^0 and f_s^0 represent the aspiration levels of criteria functions that will be achieved by the decision maker.

Assuming that the membership functions based on preference or satisfaction are linear, we can present the linear membership functions for criteria functions and constraints as follows:

$$\mu_{f_r}(\underline{x}) = \begin{cases} 1 & \text{for} \quad f_r \le f_r^- \\ (f_r^+ - f_r(\underline{x}))/(f_r^+ - f_r^-) & \text{for} \quad f_r^- \le f_r(\underline{x}) \le f_r^+,\ r = 1,2,\ldots,p \\ 0 & \text{for} \quad f_r \ge f_r^+ \end{cases} \tag{15}$$

$$\mu_{f_s}(\underline{x}) = \begin{cases} 1 & \text{for} \quad f_s \ge f_s^+ \\ (f_s(\underline{x}) - f_s^-)/(f_s^+ - f_s^-) & \text{for} \quad f_s^- \le f_s(\underline{x}) \le f_s^+,\ s = p+1, p+2,\ldots,k \\ 0 & \text{for} \quad f_s \le f_s^- \end{cases} \tag{16}$$

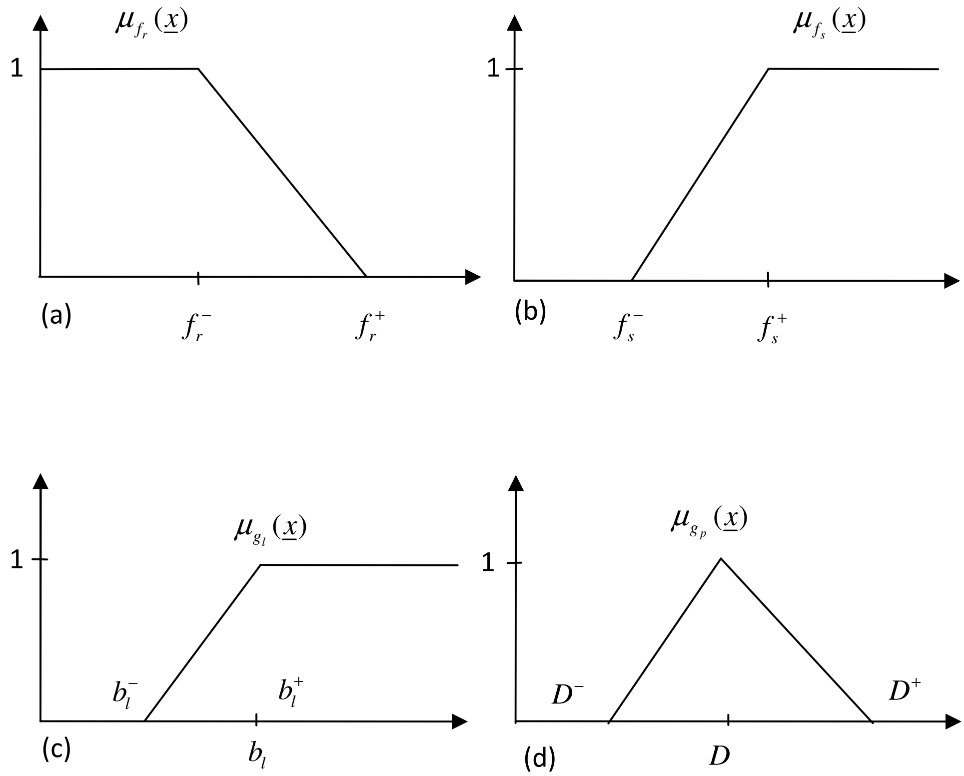
$$\mu_{g_l}(\underline{x}) = \begin{cases} 1 & \text{for} \quad g_l(\underline{x}) \ge b_l \\ (g_l(\underline{x}) - b_l^-)/(b_l - b_l^-) & \text{for} \quad b_l^- \le g_l(\underline{x}) \le b,\ l = 1,2,\ldots,m \\ 0 & \text{for} \quad g_l(\underline{x}) \le b_l^- \end{cases} \tag{17}$$

$$\mu_{g_p}(\underline{x}) = \begin{cases} 1 & \text{for} \quad g_p(\underline{x}) = D \\ (g_p(\underline{x}) - D^-)/(D - D^-) & \text{for} \quad D^- \le g_p(\underline{x}) \le D \\ (D^+ - g_p(\underline{x}))/(D^+ - D) & \text{for} \quad D \le g_p(\underline{x}) \le D^+ \\ 0 & \text{in other cases,} \end{cases} \tag{18}$$

where $b_l^- = b_l - d_l$, and $b_l^+ = b_l + d_l$, and $D^- = D - p_1$, $D^+ = D + p_2$. d_l are subjectively determined constants expressing the limits of allowed deviations of l inequation (tolerance interval) and p_1, p_2 are subjectively determined constants expressing the limits of allowed deviations of equation $g_p(\underline{x})$.

The graphic presentation of membership functions looks like this:

Figure 1. Fuzzy linear membership functions for target functions and constraints: (a) minimization of criteria functions, (b) maximization of criteria functions, (c) constraints of the type $\geq$, (d) constraint of the type $=$.



Source: own creation

In the fuzzy programming model, according to Zimmermann's approach, the fuzzy approach represents the average intersection of all the fuzzy sets that represent fuzzy criteria functions and fuzzy constraints. The fuzzy solution for all the fuzzy goals and fuzzy constraints is given as follows:

$$\mu_D(\underline{x}) = \left\{ \left\{ \bigcap_{j=1}^{k} \mu_{f_j}(\underline{x}) \right\} \cap \left\{ \bigcap_{i=1}^{m} g_l(\underline{x}) \right\} \cap \mu_{g_p}(\underline{x}) \right\}. \tag{19}$$

The optimal solution ($\underline{x}^*$) is:

$$\mu_D(\underline{x}^*) = \max_{\underline{x}\in X_D} \mu_D(\underline{x}) = \max_{\underline{x}\in X_D} \min\left[\min_{j=1,\dots,k} \mu_{f_j}(\underline{x}), \min_{l=1,\dots,m} \mu_{g_l}(\underline{x}), \min \mu_{g_p}(\underline{x}) \right]. \tag{20}$$

The optimal solution ($\underline{x}^*$) of the above model can be obtained by solving the following linear programming model (Zimmermann 1978):

$$(\max)\lambda \tag{21}$$

s.t.

$$\lambda \le \mu_{f_j}(\underline{x}), \quad j = 1,2,\dots,k \tag{22}$$

$$\lambda \le \mu_{g_l}(\underline{x}), \quad l = 1,2,\dots,m \tag{23}$$

$$\lambda \le \mu_{g_p}(\underline{x}) \tag{24}$$

$$0 \le x_i \le u_i, \quad i = 1,\dots,n; \ \lambda \in [0,1], \tag{25}$$

where $\mu_D(\underline{x})$ is the membership function for the optimal solution, $\mu_{f_j}(\underline{x})$ represents membership functions for criteria functions, $\mu_{g_l}(\underline{x})$ represents membership functions for constraints of type $\ge$, and $\mu_{g_p}(\underline{x})$ represents a membership function for constraint of type $=$. In this model the relation between constraints and criteria functions is totally symmetrical (Zimmermann 1978), and here the decision maker cannot express the relative importance of criteria functions and constraints.

In order to express the relative importance of criteria functions and constraints we have to solve the so called weight additive model in which weights present utility functions of criteria functions and constraints (Bellman-Zadeh 1970, Sakawa 1993, Tiwari et al. 1987 and Amid et al. 2006).

The convex fuzzy model proposed by Bellman and Zadeh 1970 and Sakawa 1993 and the weight additive model, by Zimmermann (1978) is

$$\mu_D(\underline{x}) = \sum_{j=1}^{k} w_j \mu_{f_j}(\underline{x}) + \sum_{l=1}^{m} \beta_l \mu_{g_l}(\underline{x}) + \gamma \mu_{g_p}(\underline{x}), \tag{26}$$

$$\sum_{j=1}^{k} w_j + \sum_{l=1}^{m} \beta_l + \gamma = 1, \quad w_j, \beta_l, \gamma \ge 0, \tag{27}$$

where w_j, β_l and γ are weight coefficients representing the relative importance between the fuzzy criteria functions and fuzzy constraints.

To solve the above fuzzy model we will use the following linear programming model:

$$(\max) f = \sum_{j=1}^{k} w_j \lambda_{1j} + \sum_{l=1}^{m} \beta_l \lambda_{2l} + \lambda_3 \tag{28}$$

s.t.

$$\lambda_{1j} \le \mu_{f_j}(\underline{x}), \quad j = 1,2,\dots,k, \tag{29}$$

$$\lambda_{2l} \leq \mu_{g_l}(\underline{x}), \quad l = 1,2,\ldots,m, \tag{30}$$

$$\lambda_3 \leq \mu_{g_p}(\underline{x}) \tag{31}$$

$$0 \leq x_i \leq u_i, \quad i = 1,\ldots,n; \tag{32}$$

$$\lambda_{1j}, \lambda_{2l}, \lambda_3 \in [0,1], \quad j = 1,2,\ldots,k; l = 1,2,\ldots,m, \tag{33}$$

2.4. Sensitivity analysis

Sensitivity analysis has to indicate robustness of the obtained solutions in vendor selection and in determination of the quantities supplied from them. After the application of FLP it is necessary to test the sensitivity of the obtained solutions caused by changes in criteria weights.

3. Case study

3.1. Criteria for vendor selection

Vendor selection and determination of quantities supplied by the selected vendors is a multicriteria problem. The most important issue in vendor selection is the choice of criteria for their evaluation. A large number of criteria that can be used in vendor selection are offered in literature. Which criteria will be chosen by the decision maker depends on the kind of problem to be solved. In this study we will consider criteria that can be used by producers of bakery products when selecting flour vendors.

Criteria used for evaluation of flour vendors can be:

- flour purchasing costs,
- flour quality, and
- vendor reliability.

Flour purchasing costs involve unit cost and transportation costs expressed in monetary units per ton.

Flour quality criteria important for bread production are expressed by the following parameters:

- General characteristics of flour (moisture in %, ash in %, acidity level ml/100 grams and wet gluten in %),
- Farinograph (water absorption %, dough development in minutes and mellowness in FU),
- Extensigraph (energy in square centimeters after 60 minutes of dough resting, elasticity in mm and resistance in extensigraph units (EU), and

- Amylograph (peak viscosity in AJ). Indicators of swelling time, temperature maximum and gluten formation time are not significant for bread production technology, therefore are not taken into account here.

It is very important to use appropriate methods for flour analysis consistently.

When contracting flour supply, it is important to find reliable vendors, i.e. those that are assumed with a high degree of certainty that will not get into financial difficulties which could result in supply discontinuation. To evaluate vendor reliability we can use indicators of their solvency, financial stability, indebtedness, liquidity, and financial performance.

Solvency indicators may be: total cash inflow in the last 30 days, average balance in the last 30 days, the amount of credit allowed, data on the number of continuous days of frozen account, number of frozen account days in the last 180 days. These data can be obtained from the SOL 2 form issued by the bank in which the vendor's main account is opened. In our opinion, it is risky to do business with suppliers with a frozen account, or with those that have had a frozen account in the last 180 days, and such vendors should be eliminated before the selection process.
Indicators of financial solvency, indebtedness, and liquidity can be: coverage of fixed assets and stocks by capital and long term resources, share of capital in source of funds in %, annual indebtedness factor, total assets turnover coefficient, general liquidity coefficient, short term receivables collection period, average sale period in days.

Indicators of financial performance are: coefficient of total revenue and expenditure ratio, share of profit in total income in %, share of profit in assets in %, and profit per employee in monetary units.

Decision maker's subjective evaluation can also be one of the indicators of vendor reliability. This indicator becomes very important in cases when company has a long standing business relationship with the vendor.

It is to be noted that it is not advisable to do business with unreliable vendors. In most cases practice shows that vendor reliability and their product quality are correlated, so that the vendors ranked high in terms of quality are usually also ranked high in terms of reliability. Indicators of solvency, indebtedness, and liquidity, as well as indicators of financial performance can be obtained from the form BON-1 issued by the central financial agency that keeps records of all legal entities based on their financial statements.

Vendor reliability criteria should include those indicators that in the period covered by the contract of purchase can have a negative effect on delivery of goods. A large number of vendor reliability indicators will make the decision making difficult. It would be hard to adequately evaluate vendor reliability without support of experts and application of quantitative methods. Therefore we will here propose quality and reliability criteria for whose application collecting data will not be a problem.

3.2.Data required for vendor selection and determination of supply quotas

We will show the example of vendor selection for a bakery. It is to be noted that in production of bread and bakery products the purchase of flour is contracted for the period of one year, from harvest to harvest (which usually does not correspond to the calendar year). After the harvest flour producers have the information on the available wheat quantity, price and quality which allows them to define the price, quality and quantity of flour they can supply in the subsequent one-year period.

In the one-year period the bakery plans to consume 4000 tons of flour Type 550. The company contacts 4 potential flour suppliers and defines the upper limit of flour supplied by a single vendor in the amount of 1500 tons. The proposed prices of flour and transportation costs (Criterion C1) are shown in the Table 1. The potential vendors supply the data on flour quality that they have to maintain throughout the contract period (Criterion C2). It is to be noted that the quality of flour depends on the wheat sort and quality and on technology used in flour production. The vendors also should supply data on their reliability by the forms SOL-2 and BON-1 (Criterion C3). The Tables 2 and 3 show the flour quality indicators and vendor reliability. The weights expressing the relative importance of criteria and sub-criteria are given in brackets, and they are determined by the decision maker where in every group of sub-criteria the sum of weights is 1.

Table 1. Purchasing costs for flour Type 550

Vendor	Purchasing price in MU/ton (B1)	Transportation cost in MU/ton (B2)	Total purchasing costs per ton in MU
1	2300	100	2400
2	2180	120	2300
3	2090	110	2200
4	2120	130	2250

Source: own creation

Table 2. Quality indicators fo flour Type 550

Quality indicators	Criteria weights	Vendor 1	2	3	4
General characteristics of flour (A1)	**(0.20)**				
Moisture in % (B3)	min (0.30)	13.53	13.27	13.49	13.33
Ash in % (B4)	min (0.20)	0.57	0.549	0.53	0.486
Acidity level in ml/100 grams (B5)	min (0.10)	1.5	1.5	1.6	1.8
Wet gluten in % (B6)	max (0.40)	26.7	25.8	25.1	24.0
Farinograph (A2)	**(0.30)**				
Water absorption in % (B7)	max (0.40)	60.8	59.8	58.5	61.1
Degree of mellowness in FJ (B8)	min (0.60)	70	65	85	60
Extensigraph (A3)	**(0.30)**				
Energy u cm^2 (B9)	max (0.40)	81	104	87.2	107.3
Elasticity in mm (B10)	max<190 (0.30)	137	162	180	165
Resistance (B11)	max (0.30)	395	280	235	350
Amylograph (A4)	**(0.20)**				
Peak viscosity in BU (B12)	max (1.00)	1054	860	1275	1325

Source: own creation

Table 3. Vendor reliability indicators

Reliability indicators	Criterion	Vendor 1	2	3	4
Financial stability, indebtedness and liquidity (A5)	**(0.60)**				
Coverage of fixed assets and stocks by capital and long term resources, (B13)	max (0.20)	1.12	0.88	0.87	0.92
Share of capital in source of funds in %, (B14)	max (0.10)	49.36	23.6	48.92	49.69
Indebtedness factor, number of years (B15)	min (0.10)	7	19	13	19
Total assets turnover coefficient (B16)	max (0.10)	0.65	0.49	0.52	0.35
General liquidity coefficient (B17)	max (0.30)	7.17	1.19	1.07	0.75
Short term receivables collection period, in days (B18)	min (0.20)	86	101	102	58
Performance indicators (A6)	**(0.40)**				
Coefficient of total revenue and expenditure ratio (B19)	max (0.20)	1.06	1.03	1.03	1.02
Share of profit in total income in % (B20)	max (0.30)	4.81	1.85	2.66	1.02
Share of profit in assets in % (B21)	max (0.20)	3.14	0.91	1.39	1.01
Profit per employee in mu (B22)	max (0.30)	60538	21189	12370	15446

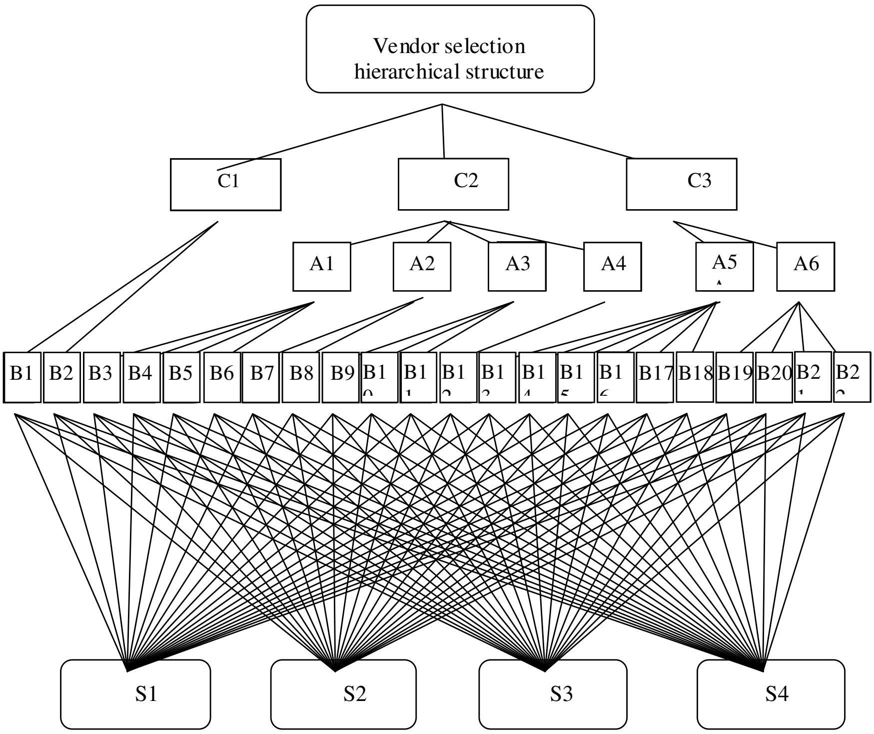
*Source:*own creation

According to the indicators from the form SOL-2 potential vendors have an active current account that has not been frozen in the last 180 days.

3.3. Application of revised weighting method

Considering the data from the Tables 1, 2 and 3 we form a hierarchical structure of goals and criteria for vendor selection. The hierarchical structure is shown in the Figure 2.

Figure 2. Hierarchical Structure of Suppliers Selection



Source: own creation

The hierarchical structure in our example consists of five levels as shown in the Figure 2. Level 1 represents the vendor general efficiency (or total value of purchasing - TVP), Level 2 represents criteria for vendor selection, Level 3 represents criterion sub-criteria, Level 4 represents sub-criterion sub-criteria, and Level 5 represents the available alternatives (vendors).

After decomposition of the problem and formation of the hierarchical structure of goals and criteria, we have applied a revised weight coefficients method to calculate the coefficients of cost, quality and reliability functions. By application of the relation (3) and (5) the cost function coefficients are normalized. The following weights are obtained:

Table 4. Normalized coefficients of cost function

Variable	Coeff. c'_{i1}	Coeff. c''_{i1}
x_1	1	0.262295
x_2	0.958333	0.251366
x_3	0.916667	0.240437
x_4	0.9375	0.245902

Source: own creation

The quality function has a hierarchical structure and has to be maximized. Sub-criteria B3 to B12 are grouped into 4 sub-criteria sets. According to the data on coefficients weights, their linear transformation and normalization into the interval $[0,1]$ is carried out. The normalized coefficient values are shown in the following table:

Table 5. Normalized coefficient values with variables for sub-criteria B3-B12

Var.	c''_{iB3}	c''_{iB4}	c''_{iB5}	c''_{iB6}	c''_{iB7}	c''_{iB8}	c''_{iB9}	c''_{iB10}	c''_{iB11}	c''_{iB12}
x_1	0.247674	0.233287	0.265193	0.262795	0.253122	0.245874	0.213439	0.212733	0.313492	0.233496
x_2	0.252527	0.242211	0.265193	0.253937	0.248959	0.264788	0.274045	0.251553	0.222222	0.190518
x_3	0.248409	0.250894	0.248619	0.247047	0.243547	0.202485	0.229776	0.279503	0.186508	0.282455
x_4	0.25139	0.273608	0.220994	0.23622	0.254371	0.286853	0.28274	0.256211	0.277778	0.293531

Source: own creation

Using the data on coefficient weights with variables of grouped sub-criteria and weight coefficients with sub-criteria A1, A2, A3 and A4, and by applying the relation (1) we calculate the coefficients with criterion C2 variables:

Table 6. Normalized coefficient weights with quality criterion variables

Variable	Coeff. c''_{i2}
x_1	0.244824
x_2	0.241625
x_3	0.241354
x_4	0.272198

Source: own creation

Reliability criterion coefficients are calculated in a similar way:

Table 7. Normalized coefficient weights with reliability criterion variables

Variable	Coeff. $c_{i3}^{''}$
x_1	0.397097
x_2	0.191739
x_3	0.208131
x_4	0.203032

Source: own creation

3.4. *FLP model building and solving*

As there are constraints in terms of capacity or limited quantity supplied by a single vendor, we have to form a MLP model to determine the quantities to be supplied by selected vendors. Considering the data on normalized coefficient weights with variables of cost, quality, and reliability functions, the total demand for flour in the given period and limited quantities supplied from single vendors, we form the following MLP model:
Minimization of purchasing cost

$$(\min) f_1 = 0.262295x_1 + 0.251366x_2 + 0.240437x_3 + 0.245902x_4 \tag{30}$$

Maximization of flower quality

$$(\max) f_2 = 0.244824x_1 + 0.241625x_2 + 0.241354x_3 + 0.272198x_4 \tag{31}$$

Maximization of vendor reliability:

$$(\max) f_3 = 0.397097x_1 + 0.191739x_2 + 0.208131x_3 + 0.203032x_4 \tag{32}$$

s.t.

Total needed flour quantity, limited quantities supplied, and non-negativity of variables:

$$g_1 = x_1 + x_2 + x_3 + x_4 = 4000 \tag{33}$$

$$g_2 = x_1 \leq 1500 \tag{34}$$

$$g_3 = x_2 \leq 1500 \tag{35}$$

$$g_4 = x_3 \leq 1500 \tag{36}$$

$$g_5 = x_4 \leq 1500 \tag{37}$$

$$x_1, x_2, x_3, x_4 \geq 0 \tag{38}$$

Model (29-38) is a multi-criteria linear programming model where the coefficients of the goal functions are obtained in the first stage of problem solving by application of the revised weighting method.

Model (29-38) is first solved by linear programming method optimizing separately each of the three criteria function on the given set of constraints. The results are given in the Payoff table:

Table 8. Payoff values

Solution	$(\min) f_1(\underline{x})$	$(\max) f_2(\underline{x})$	$(\max) f_3(\underline{x})$
x_1^*	980.8745	1011.953	808.4835
x_2^*	1013.662	1017.158	1091.933
x_3^*	1000.00	1001.465	1110.874

Source: own creation

It can be seen that the obtained solutions differ and that we have to choose a compromise solution. This work proposes methodology for vendor selection and determination of supply quotas by application of fuzzy linear programming on the model (29-38) in which the functions f_1, f_2 and f_3 are optimized on the given set of constraints. The application of FLP requires determination of the highest and lowest value for each criteria function. These values represent the aspiration levels in FLP. The lowest and highest values for criteria functions are shown in the following table

Table 9. Fuzzy goals

Criteria	Value-I	Value-II
f_1	980.8745*	1013.662
f_2	1001.465	1017.158*
f_3	808.4835	1110.874*

Source: own creation

Based on the above data we calculate the linear membership functions:

$$\mu_{f_1}(\underline{x}) = \begin{cases} 0 & \text{if} \quad f_1(\underline{x}) \geq 1013.662 \\ 1 - \dfrac{(f_1(\underline{x}) - 980.8745)}{(1013.662 - 980.8745)} & \text{if} \quad 980.8745 \leq f_1(\underline{x}) \leq 1013.662 \\ 1 & \text{if} \quad f_1(\underline{x}) \leq 980.8745, \end{cases} \tag{39}$$

$$\mu_{f_2}(\underline{x}) = \begin{cases} 0 & \text{if} \quad f_2(\underline{x}) \leq 1001.465 \\ 1 - \dfrac{(1017.158 - f_2(\underline{x}))}{(1017.158 - 1001.465)} & \text{if} \quad 1001.465 \leq f_2(\underline{x}) \leq 1017.158 \\ 1 & \text{if} \quad f_2(\underline{x}) \geq 1017.158, \end{cases} \tag{40}$$

$$\mu_{f_3}(\underline{x}) = \begin{cases} 0 & \text{if} \quad f_3(\underline{x}) \leq 808.4835 \\ 1 - \dfrac{(1110.874 - f_3(\underline{x}))}{(1110.874 - 808.4835)} & \text{if} \quad 808.4835 \leq f_3(\underline{x}) \leq 1110.874 \\ 1 & \text{if} \quad f_3(\underline{x}) \geq 1110.874. \end{cases} \tag{41}$$

Based on the calculated membership functions the model (29-38) can be transformed into the following linear programming model:

$$(\max)\lambda \tag{42}$$

s. t.

$$\lambda \leq \mu_{f_1}(\underline{x}) \tag{43}$$

$$\lambda \leq \mu_{f_2}(\underline{x}) \tag{44}$$

$$\lambda \leq \mu_{f_3}(\underline{x}) \tag{45}$$

$$x_1 + x_2 + x_3 + x_4 = 4000 \tag{46}$$

$$0 \leq x_1, x_2, x_3, x_4 \leq 1500 \tag{47}$$

$$0 \leq \lambda \leq 1. \tag{48}$$

The model (42-48) is a linear programming model. By solving it we obtain the following optimal solution:

$$\lambda_{\max} = 0.6708,\ \ x_1 = 987.7088,\ \ x_2 = 12.2912,\ \ x_3 = 1500,\ \ x_4 = 1500,$$
$$f_1 = 991.6692,\ \ f_2 = 1015.113,\ \ f_3 = 1011.317.$$

The fuzzy technique applied in the model (42-48) solving does not take into account the subjective importance of criteria functions. In order to include the subjective importance of criteria functions for the decision maker we solve the model (28-33), where we determine the criteria weights: $w_1 = 0.40$, $w_2 = 0.40$ and $w_3 = 0.20$. We obtain the following solution:

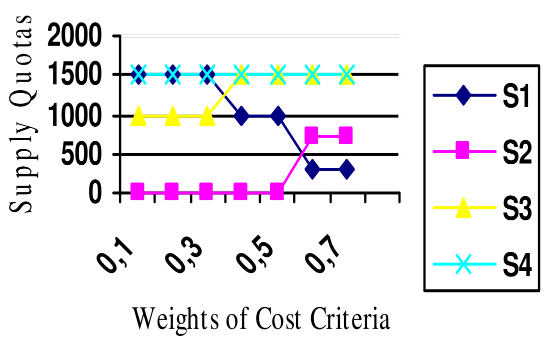
$\lambda_1 = 0.6667$, $\lambda_2 = 0.8722$, $\lambda_3 = 0.6791$, $x_1 = 1000$, $x_2 = 0$, $x_3 = 1500$, $x_4 = 1500$,

$f_1 = 991.8035$, $f_2 = 1015.152$, $f_3 = 1013.842$.

3.5. *Sensitivity analysis*

We will show sensitivity analysis of the quantities supplied by selected vendors according to changes in weights given to individual criteria by the decision maker. The selected vendors supply quotas as the consequence of increased weights in purchasing cost criterion with reduced weight in product quality criterion and keeping the weight in vendor reliability criterion at the level of 0.20 (case I) are shown in the following figure:

Figure 3. Vendors' supply quotas (case I)

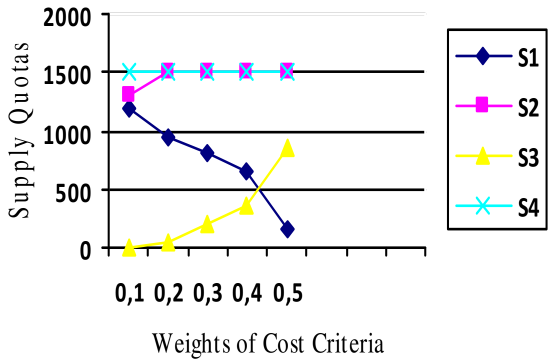


Source: own creation

It is obvious that the supply quota from the vendor S4 does not change no matter what the increase of cost criterion importance is which means that the vendor S4 is not sensitive to changes in cost criterion with simultaneous decrease in the quality criterion. The supply quota from this vendor remains 1500 t, which shows that the vendor S4 is the best in terms of both cost and quality criteria. The increase of importance of the cost criterion from 0.10 to 0.50, with simultaneous decrease of importance quality criterion from 0.70 to 0.30 does not change the supply quota from the vendor S2 (it remains zero). However, the increase in importance of the cost criterion from 0.50 to 0.60 with simultaneous decrease of the quality criterion importance from 0.30 to 0.20 causes a significant increase of supply quota from this vendor, from 0 to 704.5 tons. The vendor S1 is negatively sensitive to the increase of cost criterion importance in the interval of 0.30 to 0.70, while the vendor S3 is positively sensitive to the increase of cost criterion importance from 0.30 to 0.40, whereby further increase of cost criterion importance does not affect this vendor's supply quota. The vendors S1 and S3 are not sensitive to the increase of cost criterion importance from 0.10 to 0.30, while the vendor S2 is not sensitive to the increased importance of the cost criterion from 0.10 to 0.50.

It is interesting to observe the changes in supply quotas caused by the changes in weights of cost and reliability functions with the constant weight of 0.40 for quality function (case II). The graph presenting these relations is shown in the Figure 4.

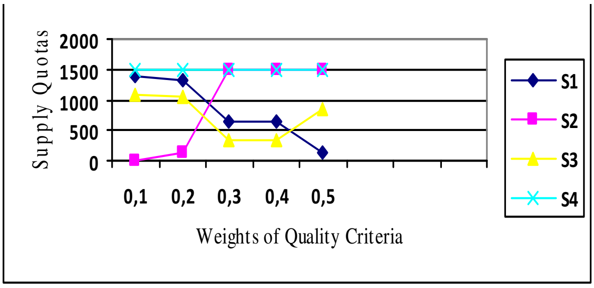
Figure 4. Vendors' supply quotas (case II)



Source: own creation

Changes in supply quotas caused by changes of weights in quality and reliability functions with the constant cost function weight 0.40 (case III), is shown in the following figure:

Figure 5. Vendors' supply quotas (case III)



Source: own creation

Observing the above figures we can conclude that the vendor S4 is not sensitive to changes in criteria functions importance coefficients. As this vendor is the best in terms of all the criteria we should consider increasing the supply quotas from this vendor. The supply quota from S2 increases from zero to 852.75 tons and in the case when the weight of cost function rises from 0.40 to 0.50 with the weight of quality function remaining at 0.40 and the weight of reliability function dropping from 0.20 to 0.10. In all the other cases this vendor's supply quota is equal to zero, which makes us conclude that we should avoid purchase from this vendor. The supply quotas from vendors S1 and S3 are sensitive to changes in criteria functions importance. However, the vendor S3 is positively sensitive to changes in criteria functions weights, which suggests that we should consider the possibility of increasing the supply quota from this vendor.

4. Conclusion

Solving the concrete example by application of the proposed methodology we can make a number of conclusions presenting the advantages of using the revised weighting method and FLP in comparison to the application of only one of them.

The revised weighting method allows efficient reducing of complex criteria functions into simple criteria functions. For DM, it is easier to determine weighting coefficients if he/she deals with few criteria functions than if he/she deals with a large number of them. If there are a large number of criteria and sub-criteria, there is a high probability of error in determining of weighting coefficients.

The weight coefficient's method applied alone has some shortcomings so that it is not the most appropriate one to create a set of Pareto optimal solutions. The shortcomings are: (1) varying weight coefficients do not guarantee that we will determine all Pareto optimal solutions, and (2) the determined Pareto optimal solutions are those that are situated in the extreme points of the convex polyhedron but not those that connect the two extreme points. To determine the set of compromise solutions and the preferred solution it is better to use the fuzzy linear programming model.

When solving the MLP model the use of fuzzy technique proves to be very efficient. The efficiency of the fuzzy technique in solving the model can be seen in the possibility to define weights for criteria functions that express the decision maker's preferences. However, if you deal with complex criteria functions it is complicated to use the FLP method alone because of arising problems by determination of weighting coefficients.

Application of revised weighting method and FLP to solve the problem of vendor selection and determination of supply quotas allows a simple sensitivity analysis of the obtained solutions. The proposed methodology can be used in solving similar business problems.

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