

Floating utility criterion in a problem of optimal budget distribution

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This paper contains the research of neuroeconomics results such as formulation and analysis of Ultimatum game (Sanfey et al. 2003) and neuromarketing (Renvoisé-Morin 2007). As a result the rational behavior of consumer during the decision-making of consume object prejudiced. In particular the axiom of reflexivity of the rational utility theory was disproved. That axiom maintains that the fixed set of goods is not worse than itself. A conclusion that consumer choice based on the utility criterion depends not only on the set of goods but on the consume environment was made. The hypothesis of irrational behavior allowed to formalize floating utility criterion and correlation between the basket of products utility and consume environment during the consumer decision-making. Based on floating utility criterion the problem of optimal consumer's budget distribution in conditions of integral utility maximization on limited time interval and consideration of the predicted environment factors value posed. Then the problem of intertemporal consumer choice for floating criterion was posed. The solution analysis of that problems had allowed to draw a conclusion of a significant influence of the predicted environment factors value exactness on an optimal solution and a dependence of that exactness on a consumer satisfaction.

Keywords: utility, floating criterion, decision-making, optimality

1. Introduction

Rationality of classical economic theory is often questioned. Different areas of behavioral economics give different interpretation of the rationality of individual decision-making.

Risk theory formulates such irrationality in the form of the Allais paradox, the effects of equal treatment and equal difference, is trying to formalize the heterogeneity of individual behavior with the factor called "propensity for risk".

Neuroeconomical science, standing at the crossroads of economics and neuropsychology, explores the reasons of the economic decision-making depending on the activity of various parts of the brain. The origin of this area of science came through the formulation and investigation of the "Ultimatum game", as proposed in 2003 by Alan G. Sanfey in his article "Neural basis of decision-making in the

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ultimatum game» (Sanfey et al. 2003). Meaning of the Game is: one of the participants must share, for example, \$10 for himself and the second party, which means he can offer any amount from \$0 to \$10. The second party, after a proposal by a certain amount, must decide to accept it or not. If he agrees to take this amount of money, both parties remain the same amount of money, which they agreed. If the second party refuses, then they both remain with nothing. Rational economic theory suggests that any offer greater than zero must be taken because any positive amount of money is better than nothing. Empirical results showed that offers with amount of the \$ 2 or \$ 3 was rejected. Researchers of the game found a correlation between the refusal of the second party, during an unfair division, and activation of insula - the brain area associated with negative emotions. Also, they found a correlation between the agreement to accept the amount and activation in the dorso-lateral prefrontal cortex, which is connected with our thinking and planning actions.

One example of the capitalization of knowledge about the irrationality of decision-making is Neuromarketing (Renvoisé-Morin 2007) - a purely practical discipline that studies the decision-making to purchase consumer products during an influence of a variety of emotional factors, such as advertising, packaging, etc.. Illustration of correlation between the certain decisions and the emotional influence can be shown by the example of the American consumer, which in terms of consumption utility sees no difference between Coke and Pepsi. This result was obtained during the experiment, in which consumers were blindfolded and drank the appropriate drinks, while he did not know which one he is drinking at the moment. The activity of the brain during the consumption of different beverages were indistinguishable. If the consumer knew that he is drinking Coke at the moment, the utility which he received, expressed in activity in his brain parts was more than the utility of Pepsi by 10 times. This effect is achieved through aggressive advertising of the first drink and its other characteristics not related to the ingredients of product consumed. The result of this study is allowed not to speak on the formed consumer preferences, which described by mainstream marketing and studied in a rational utility theory, but a preference, depending on the consumption context. The consumption decision-making depends not only on the product itself, but also on external conditions under which consumption takes place. Thus, a certain set of products under different external conditions may be different levels of consumer utility. This statement refutes the first axiom of rational preferences (the axiom of reflexivity - a specific set of goods is not worse than himself) and gives rise to further study and formalization of floating utility criterion.

2. Problem of optimal budget distribution

Assume that the utility function of rational consumer is given in standard form $U: R^n \rightarrow R^1$, $X = \{x_i\}$, $i = \overline{1, n}$, $U(X) = U(x_1, \dots, x_n)$. On the utility of

consumption of these products have a significant influence m environmental factors. Let specify that factors as $S = \{s_j\}, j = \overline{1, m}$. The influence is specified in the form of the operator $K: R^n \rightarrow R^{n+m}$ or $K = \{k_{ij}(s_j)\}$, where $k_{ij}(s_j)$ is the functional coefficient of the utility sensitivity of the good x_i to a external factor s_j . Thus, the utility function takes the form: $U(K[X]) = U(X, S) = U(\{(\prod_{j=1}^m k_{ij}(s_j))x_i\})$ for the case of the multiplicative effect of environmental factors on the utility from consumption of the product. In this case, if the factor s_j has no effect on the good x_i , then the value will be $k_{ij} = 1$ for every value of s_j . Utility function, written in this form will be called a floating utility criterion of the consumption basket of products.

Consider the application of this function to the problem of the optimal budget distribution - this is the problem of optimal distribution of the consumer funds in terms of maximizing the integral utility of consumer basket for a limited time interval.

Utility function is given in the form $U(\{\prod_{j=1}^m k_{ij}(s_j)x_i\})$. There is $x_i = x_i(t)$ – consumption of the i -th product at time t and $s_j = s_j(t)$.

$I(t)$ – the cash flow at time t .

$I(t) \geq \sum_{i=1}^n C_i(t)$ – budget restriction, where $C_i(t) = p_i(t)x_i(t)$ – the cost of consumption of the i -th product. In this problem defined condition non-borrowing and non-crediting of funds period t for other periods.

The solution of the problem requires a description of the matrix K , in the form of a rectangular matrix $m*n$ functional coefficients of the utility sensitivity of the consumer goods i from the external environmental factors j . Methods of specification of functional coefficients are not listed, but they may be obtained from the research methods of neuroeconomics, the theory of risk, the theory of adaptive preferences, expert - it depends on the subject area of the problem and of tools explorer.

Another important parameter of the problem are the values of functions $s_j(t)$. Assume that they are known, or set a forecast value for the period T .

The last parameter required to solve the problem is the set of forecast of the dynamics of prices of products $p_i(t)$.

Thus, the problem becomes:

$$\int_0^T U(\{\prod_{j=1}^m k_{ij}(s_j(t))x_i(t)\}) dt \rightarrow \max (*)$$

$$I(t) \geq \sum_{i=1}^n C_i(t)$$

$$x_i(t) \geq 0, \forall i = \overline{1, n}, \forall t \in [0, T]$$

Consider the solution to this problem (*) a certain example:

$$U(X) = a_0(a_1x_1^{\alpha_1} + a_2x_2^{\alpha_2})^\beta$$

$$a_0 = 1, a_1 = 2, a_2 = 3, \alpha_1 = 0.3, \alpha_2 = 0.6, \beta = 1$$

$$K = \begin{pmatrix} -0.002s_1 + 1.5 & 1 \\ 1 & -0.01s_2 + 0.01e^{s_2} \end{pmatrix}$$

$$S = \begin{pmatrix} 180 + 180\sin\left(\frac{2t}{\pi}\right) \\ 1 + 0,6t \end{pmatrix}$$

$$i(t) = 2001\ln(t + 1) + 500$$

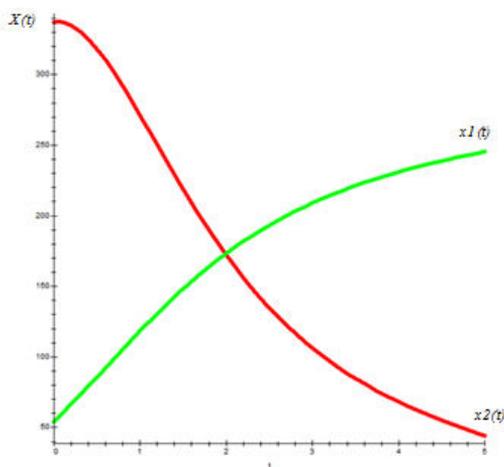
$$P(t) = \left(\left\{ p_i^0 \left(1 + \frac{t}{T} \pi_T \right) \right\} \right)$$

$$T = 5, \pi_T = 0.1, p_1^0 = 1, p_2^0 = 3$$

Solution to this problem is reduced to the optimization of the utility function for every t (in accordance with the additivity property of the integral) in the restriction to the budget equality to costs, because that is clear that the integral utility value will be optimal in case of Pareto-efficient budget distribution at each time moment.

Thus, the solution of the problem will be the vector of functions $F(t) = \{f_i(t)\}$, defined on the interval $[0, T]$, reflecting the optimal trajectory of the consumption basket of goods for each t . The solution of the problem (*) looks like:

Figure 1. Solution $X(t)$ for the problem (*)



Source: own creation

The optimum value of accumulated utility for the period $[0, T]$ is $\int_0^T U(t) dt = 147,8171$.

Consider the sensitivity of the problem (*) solution to the exactness of the forecast trajectory set of the external environmental factors.

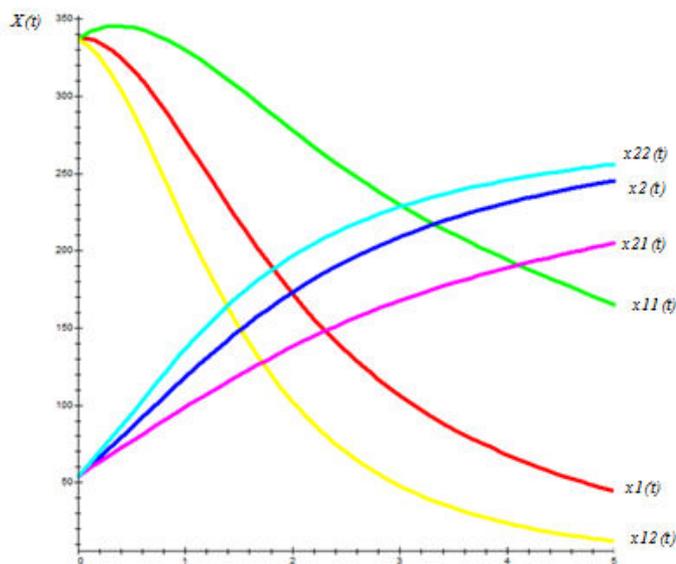
Define the forecasts of the trajectory \tilde{S} , different from S :

$$\tilde{S}_1 = \begin{pmatrix} 180 + 180\sin\left(\frac{2t}{\pi}\right) \\ 1 + 0,3t \end{pmatrix}$$

$$\tilde{S}_2 = \begin{pmatrix} 180 + 180\sin\left(\frac{2t}{\pi}\right) \\ 1 + 0,9t \end{pmatrix}$$

Obtain the optimal solution found in the assumptions of correct prediction. This decision will characterize the optimal trajectory of the consumption basket of goods in the sense of a set exactness of the forecast. These trajectories will look like: $X(t)$ $x1(t)$ $x2(t)$ $x12(t)$ $x21(t)$ $x11(t)$ $x22(t)$

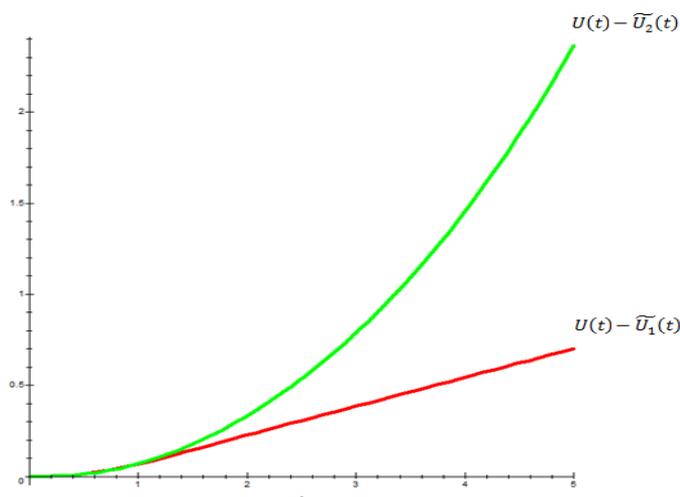
Figure 2. Sensitivity analysis of solutions $X(t)$



Source: own creation

Based on the obtained solution it seems to be possible to give a conclusion that the decision-making on the basis of not exact prediction leads to a deviation value of basket of goods utility received as predicted from the optimum obtained for the actual values of the context of consumption:

Figure 3. The deviation from the optimal solution for predictive



Source: own creation

Thus, the need to specify the forecast values of environmental factors most approximate to the reality increases the planning exactness of the trajectories of consumption and minimizes the deviation from the optimal solution.

3. Problem of utility maximization for intertemporal choice

Consider a modification of the problem (*) for a model of intertemporal choice, i.e. the possibility of borrowing funds from the budget of period t in period τ . Formally, it will look like in the period t there is the funds receiving in size $I(t)$, function $\alpha(t, \tau)$ describes the proportion of funds allocated for implementation of consumption in the period τ , then the remaining funds will amount to:

$$\tilde{I}(t) = \int_{\tau=0}^T \alpha(\tau, t) (1 + \frac{r}{T} |t - \tau|)^{t-\tau} I(\tau) d\tau$$
, where r is the money market interest rate.

So, for the model of the intertemporal choice, integral utility maximization problem becomes:

$$\int_0^T U(\{\prod_{j=1}^m k_{ij}(s_j(t)) f_i(t)\}) \rightarrow \max (**)$$

$$\tilde{I}(t) \geq \sum_{i=1}^n c_i(t)$$

$$x_i(t) \geq 0, \forall i = \overline{1, n}, \forall t \in T$$

$$\int_{\tau=0}^T \alpha(t, \tau) d\tau = 1, \forall t \in T$$

The problem (***) is trivial in the sense that the optimal solution will be achieved only if

$$\alpha(t, \tau) = \alpha^*(\tau, t) = \begin{cases} \infty, & t = t^* \\ 0, & t \neq t^* \end{cases}, \text{ where } t^* \text{ is determined from the condition} \\ U(t^*) = \max_{t \in T} U(X, S, I)$$

Simply put, the budget constraint is determined by the condition of crediting all income received in the period t^* in which the maximum utility value at the point achieved.

Thus, $\hat{I}(t)$ takes the form

$$\hat{I}(t) = \left(\int_{\tau=0}^T t(\tau) \left(1 + \frac{r}{T} |t - \tau| \right)^{t-\tau} d\tau \right) \varphi(t - t^*), \text{ where } \varphi(t - t^*) = \begin{cases} 1, & t = t^* \\ 0, & t \neq t^* \end{cases}$$

The problem solution will look like $X^*(t^*)$, where:

$$\sum_{i=1}^n p_i(t^*) X_i(t^*) = \hat{I}(t^*) (I) \\ U(X^*(t^*)) = \max_{X(t^*) \in (I)} U(X(t^*))$$

Consumption of goods will occur only during the period t^* , in the remaining periods of the interval T consumption is equal to 0. Obviously that pose the problem in this form does not make sense as to sustain an individual life requires a specific set of benefits, other than 0, which yields the minimum required utility. In this situation, it makes sense to introduce in the problem (***) a restriction on the minimum required value \underline{U} , then the problem becomes:

$$\int_0^T U(\{\prod_{j=1}^m k_{ij}(s_j(t)) x_i(t)\}) dt \rightarrow \max (***)$$

$$\hat{I}(t) \geq \sum_{i=1}^n C_i(t) \\ x_i(t) \geq 0, \forall i = \overline{1, n}, \forall t \in T \\ \int_0^T \alpha(t, \tau) d\tau = 1, \forall t \in T \\ U(t) \geq \underline{U}(t), \forall t \in T$$

Consider the solution for the problem (***):

To maximize the integral utility required to provide the interval T consumption \underline{U} , respectively, to solve the auxiliary problem in terms of $\underline{x}_i(t)$ where

$$\underline{I}(t) = p_i(t) x_i(t):$$

$$\underline{I}(t) \rightarrow \min, \forall t \in T \\ U(\underline{x}_i(t)) \geq \underline{U}(t), \forall t \in T$$

$$\underline{x}_i(t) \geq 0, \forall i = \overline{1, n}, \forall t \in T$$

The obtained solution of the problem to minimize the budget will be used for solving the problem of maximizing the integral utility in terms of $\overline{x}_i(t)$ and $\overline{\alpha}(t, t)$ with the restriction on the minimum utility value, then the problem (***) becomes:

$$\int_0^T U(\{\prod_{j=1}^m k_{ij}(s_j(t))(\overline{x}_i(t) + \underline{x}_i(t))\}) dt \rightarrow \max (***)$$

$$\overline{I}(t) \geq \sum_{i=1}^n c_i(t)$$

$$\overline{I}(t) = \int_{\tau=0}^t \overline{\alpha}(\tau, t) (I(\tau) - \underline{I}(\tau)) (1 + \frac{r}{T} |t - \tau|)^{t-\tau} d\tau$$

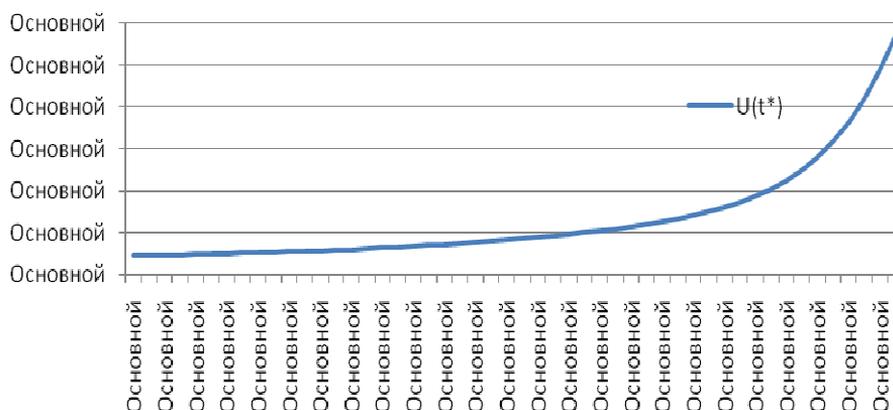
$$\overline{x}_i(t) \geq 0, \forall i = \overline{1, n}, \forall t \in T$$

$$\int_{\tau=0}^T \overline{\alpha}(t, \tau) d\tau = 1, \forall t \in T$$

The problem (***) solution is analogous to the solution of the problem (**) and will set consumption only at the point t^* , the remaining points of the interval T consumption is equal to 0. The general solution will be $\underline{x}_i(t) + \overline{x}_i(t)$, the optimal value of the integral value will be a $U(\{\underline{x}_i(t) + \overline{x}_i(t)\})$. Thus, the final solution allows to achieve the minimum required value, while at t^* will be observed jump in accumulated utility to a value of $U(\{\underline{x}_i(t^*) + \overline{x}_i(t^*)\})$

For the problem (**) found the optimal consumption for each t in the case of lending of all incoming funds in period t^* . For each solution is calculated accumulated value on the interval $[0, T]$:

Figure 4. Graph of the solution of (**) in the form of $U(t^*)$



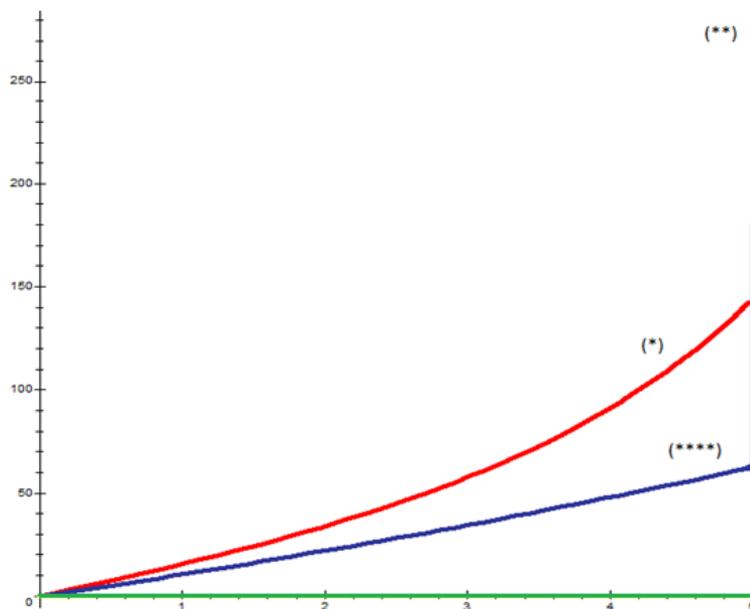
Source: own creation

Is thus seen that the maximum accumulated value is achieved if all the funds will be used for consumption in period $t^* = T = 5$. This is due to the budget growth rate for $t \rightarrow T$ and the growth rate for K , which is anticipatory for growth rate of prices. The value of accumulated utility for the problem (**) optimal solution was $\int_{t=0}^{t^*} U(t, \hat{I}(t)) dt = 285,0304$.

Founded the solution for the problem (****) in terms of problem (1) with set of constraint for $\underline{U}(t) = 10 + t$. The solution in the form of accumulated utility will be a $\int_0^T U(\{x_i(t^*) + \bar{x}_i(t^*)\}) = 181,5412$.

Consolidated graph of the solutions of problems (*), (**), (****) in the form of accumulated integral utility for the period t :

Figure 5. Comparative analysis of solutions of problems (*), (**), (****) in the form of an accumulated utility



Source: own creation

Thus, can be made a conclusion that the optimal value achieved in the problem (**) but the problem with the minimal utility value restriction most approximates the reality in studied problem class and the optimal solution is achieved in the restriction in the minimal required utility.

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