Development of the statistical process control methods

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Statistical process control represents a statistical procedure using control charts to see if any part of production process is not functioning properly and could cause poor quality. Process control is achieved by taking periodic sample from the production process and plotting these sample points on a chart, to see if the process is within statistical control limits. Statistical process control prevents quality problems by correcting the process before it starts producing defects. This paper encompasses an application of statistical process control methods with numerous modifications in order to make possible appropriate process quality improvement of the soft drink production line via detecting out-of-control process or unusual patterns in a sample. Used methods corresponding with the total quality management require a never-ending process of continuous improvement that covers people, equipment, suppliers, materials and procedure. The end goal is perfection, which is never achieved but always sought.

Keywords: Statistical process control, total quality management, control charts

1. Introduction

Quality can be described as the most important factor in the long-term profitability and success of the firms. Therefore, it can not be underestimated or overlooked by any firm, regardless of its size or assets.

Globalization and foreign competition changed the business environment and created higher expectations for products and services. Quality not only allows for product discrimination, it also has become a marketing weapon focusing on the consumer as the most important part of the production line (W. Edwards Deming) (Russel-Taylor 1998). This results in a commonly used definition of quality as a service's or product's fitness for intended use. At the other side, we assume another perspective known as producer's quality perspective which means a quality during

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production focuses on making sure that the product meets the specification required by the design. Perhaps, the most important role in achieving and improving required quality level play employees throughout the direct involvement in the management process referred to as participative problem solving. A quality circle is one of the most commonly used employee-involvement program and presents a small voluntary group of employees and their supervisors. Using team approach, it represents the brainstorming to generate ideas as a group technique for identifying and solving problems.

2. Total quality management

To make sure that products and services have the quality they have been designed for, a commitment to quality throughout the organization is required. Total quality management represents a set of management principles that focus on quality improvement as the driving force in all functional areas and at all levels in a company. However, the commitment to quality must begin at the top and spread down through the organization. Although it is popular to say that quality is everyone's responsibility in a company, total quality management generally requires a total commitment from the top management to monitor and maintain quality throughout the organization.

3. Statistical process control

A major topic in statistical quality control is statistical process control. It represents a statistical procedure using control charts to see if any part of production process is not functioning properly and could cause poor quality.

Walter Shewhart developed the technical tools that formed the beginning of statistical quality control. He and his colleagues introduced the term quality assurance as a commitment to quality throughout the organization using statistical quality control methods first time applied at Bell Telephone Laboratories. Later on W. Edwards Deming changed the focus of quality assurance from the technical aspects to more of a managerial philosophy. Today US companies that have been successful in adopting Total quality management concept train all their employees in statistical process control methods and make extensive use of statistical process control for continuous process improvement. Through the use of statistical process control, employees are made responsible for quality in their area, to identify problem and either correct them or seek help in correcting them.

Process control is achieved by taking periodic sample from the production process and plotting these sample points on a chart, to see if the process is within statistical control limits. Statistical process control prevents quality problems by correcting the process before it starts producing defects.

All processes contain a certain amount of variability that makes some variation between units inventible. There are two reasons why a production process might vary. The first is the inherent random variability of the process, which depends on the equipment, engineering, the operator and the system used for measurement. This kind of variability is a result of natural occurrences. The other reason for variability is unique causes that are identifiable and can be corrected. These causes tend to be nonrandom and, if left unattended, will cause poor quality.

4. Production quality improvement

This chapter encompasses an application of statistical process control methods in order to make possible appropriate process quality improvement via detecting out-of-control process or unusual patterns in a sample.

Total quality management requires a never-ending process of continuous improvement that covers people, equipment, suppliers, materials and procedure. The end goal is perfection, which is never achieved but always sought.

4.1. Quality measure and sample size determination

First concern of quality management after they noticed a potential quality problem is choosing a right quality measure. The quality of a product or a service can be evaluated using either an attribute or a variable measure. An attribute is a product or a service characteristic that can be evaluated with a discrete response. Attributes can be evaluated quickly and represents a qualitative classification. Even if quality specifications are complex and extensive, a simple attribute test might be used to determine if a product is or is not defective. A variable measure is a product or a service characteristic that is measured on a continuous scale. Because it is a measurement, a variable classification typically provides more information about the product.

Difference between attribute and variable measure requires different sample size determination. In general, larger sample sizes are needed for attribute charts because more observations are required to develop usable quality measure. Variable control charts require smaller sample sizes because each sample observation provides usable information. After only a few sample observations, it is possible to compute a range or a sample average that reflect the sample characteristics.

It may also be important that the samples come from a homogeneous source so that if the process is out of control, the cause can be accurately determined. If production takes place on either one of two machines, mixing the sample observations between them makes it difficult to ascertain which operator or machine caused the problem. If the production process encompasses more then one shift, mixing the sample observation between shifts may make it difficult to discover caused a quality problem in the process.

Example presented in this paper contains weight as a variable measure in monitoring quality of soft drink production process. This variable contains not only control limits from product design specifications, but also natural variations that can not be exceeded by the actual products. Natural variations known as design tolerances are design or engineering specifications reflecting customer requirements for a product. In Serbia the Regulation in quality and other requirements for soft drink products is standardized by the Serbian customer society. The mentioned Regulation provides natural tolerances for different weights of product; for example product in 1 liter package has 2% deviation and a product in 2 liter package has 1,5% deviation allowed. Considering all above mentioned facts we choose one production line and one work shift in this analysis to create numerous samples where each contain 5 actual products. Every production process has a different probability that shift in the process will be detected known as $\alpha = (1 - \beta)$, or probability of not detecting process quality shift β. β depends on the sample size, population variance σ and difference between actual mean μ_t and hypothesized mean μ_0. This coefficient presents the input value in calculation of ARL (average run length) as the expected number of samples to be taken before control chart indicates a shift in a process level.

Probability of not detecting process quality shift can be calculated using normal distribution probability formulae as follows:

$$\beta = P\left\{z < (\mu_0 - \mu_t)/(\sigma/\sqrt{n}) + z_\alpha\right\}$$

To calculate the number of samples necessary to detect some shifts it can be used the formulation below:

$$ARL = \frac{1}{1-\beta}$$

The presented production line has input values $\mu_0 = 1$, $\mu_t = 0{,}998$, $\sigma = 0{,}01$, $n = 5$. So this part of process has its own probability of not detecting shift and average run length according to the stated input values.

$\beta = P(z < 2{,}0922) = 0{,}9818$

ARL = 54,95

There is a very high percentage of not detecting shift level probability in samples. Hence average run length needs to be high and exceed the number of 54,95 samples. According to the level of ARL, quality examination of production line will use 60 equal samples taken in the same period interval from the same production line.

4.2. Control charts

Walter Shewhart developed a simple but powerful tool to separate two or more items known as the control chart. Control charts represent a basic statistical process control tool to measure performance of a process. Control charts are used to investigate the variability of the process and this is essential when assessing the capability of a process. Data are often plotted on a control chart in the hope that this may help to find causes of problem. These charts visually show if a sample is within statistical control limits. Control limits are the upper and lower bands of a control chart.

There are four basic types of control charts, two for attribute measure (p-chart and c-chart) and two for variable measure (mean-chart and R-chart). This chapter, as it is noticed before, contains the charts for variable measure. Beside the two basic types, this paper contains development of different modification in order to spot the smaller shifts in a process. However, looking at the figures alone will not give the reader any clear picture of the safety performance of the business.

4.2.1. R-chart

In this chart, the range is the difference between the smallest and largest value in a sample. This range reflects the process variability instead of the tendency toward a mean value. Control limits in this type of control chart are presented via formulas as fallows:

$$UCL = D_4 \cdot \overline{R} \quad LCL = D_3 \cdot \overline{R}$$

$\overline{R}$ is the average range, and also a central line of the chart, for the samples:

$$\overline{R} = \frac{\sum_{i=1}^{k} R_i}{k}$$

where k is the number of samples and D_3 and D_4 are factors for determining control limits that have been developed based on range values rather then standard deviations.

Figure 1. Range control chart

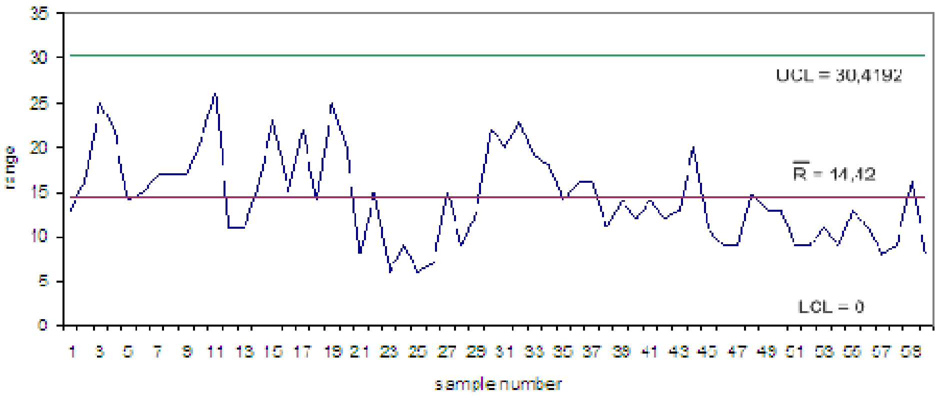

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Figure 1 indicates that the production process is in the control. There are no significant differences between ranges in each sample. Therefore, the variability observed is a result of natural random occurrences. Although individual values are all different, as a group they form a pattern that can be described as a normal distribution.

4.2.2. Mean chart

The mean of each sample is computed and plotted on the chart, so the points are the sample means. The central line of the chart is the overall process average, the sum of the averages of k samples:

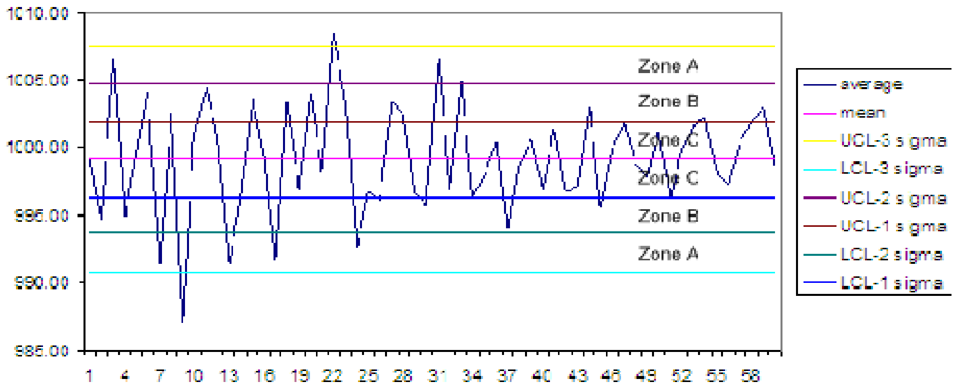
$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}$$

When the mean-chart is used in conjunction with an R-chart, the allowing formulas for control limits are used:

$$UCL = \bar{\bar{x}} + A_2 \cdot \bar{R} \quad LCL = \bar{\bar{x}} - A_2 \cdot \bar{R}$$

Where A_2 is a tabular value used to establish the control limits. Values of A_2 were developed specifically for determining the control limits for mean-chart and are comparable to 3-standard deviation limits.

Figure 2. Mean chart and zones for pattern test



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Obviously, there are two sample averages that are outside of specified control limits. Here we have a situation that the process averages are not in control like variability presented in Figure 1. This example illustrates the need to employ the R-chart and mean-chart together. In the R-chart none of the ranges for the samples were close to the control limits. However, ranges for some samples were relatively narrow, whereas means of those samples were relatively high. Hence the use of both charts together provided a more complete picture of the overall process variability.

If it is important to take a closer look on a control chart and to decide if pattern is nonrandom or random the pattern test is in wide use. One type of pattern test divides the control chart into three zones on each side of a center line, where each zone is one standard deviation wide. Figure 2. shows the numerous points (samples) in zones A and B (specially in the first part of samples) imply that nonrandom pattern exists and the cause should be investigated.

This kind of process pattern needs to be investigated using just the mean-chart with a standard deviation and the following formulas for establishing control limits are:

$$UCL = \overline{\overline{x}} + z \cdot \sigma_{\overline{x}} \;,\; LCL = \overline{\overline{x}} - z \cdot \sigma_{\overline{x}}$$

The sample standard deviation is computed as:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

Figure 3. Mean chart using standard deviation

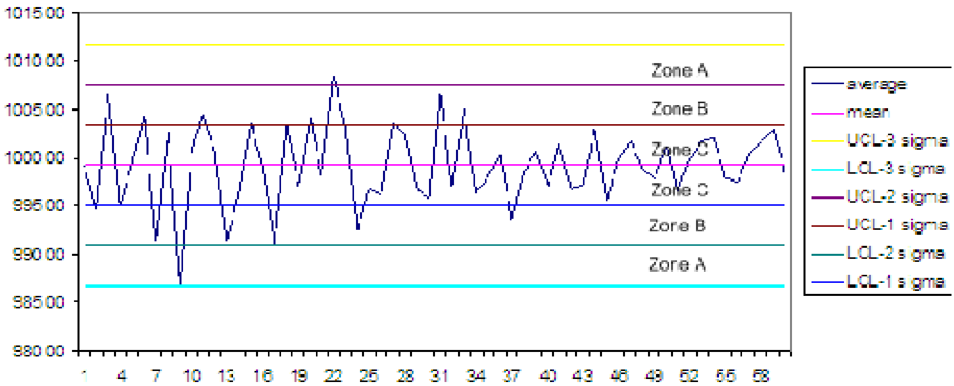

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Figure presented above shows corrected control limits according to rule of 3σ. After the correction there are no samples beyond control limits, but there still exists a nonrandom pattern of used samples because of numerous sample averages in zones A and B. The points plotted in zone C, also known as green zone, show stability in a process. The sample points plotted in zone B (yellow zone) suggests there may have been an assignable change and that another sample must be taken in order to check. Any points plotted in zone A (red zone) indicate that the process should be investigated and that, if action is taken, the latest estimate of the mean and its difference from the original process mean or target value should be used to assess the size of any correction.[3]

Interpretation of the nonrandom pattern presented through the shift in process level may be caused by the introduction of new production process or new workers, changes in methods, row materials or machines, a change in the inspection methods etc. The presented time series are stationary and uncorrelated according to the augmented Dickey-Fuller test of unit root (ADF = 7,1379 > $\chi^2(1\%)$ = 3,5457, the null hypothesis is rejected and production process sample series is a stationary process) and autocorrelation and partitial autocorrelation functions are not showing any specified model formulation. They are showing that the process is in control and stabile.

The control process suspected to posses a nonrandom pattern needs to be investigated by modified control charts. These control charts are useful when it is important to detect small shift in a process.

4.2.3. Modified control charts

[3] Oakland, John (2003) Statistical Process Control – fifth edition, Butterworth – Heinemann, Oxford, p 120

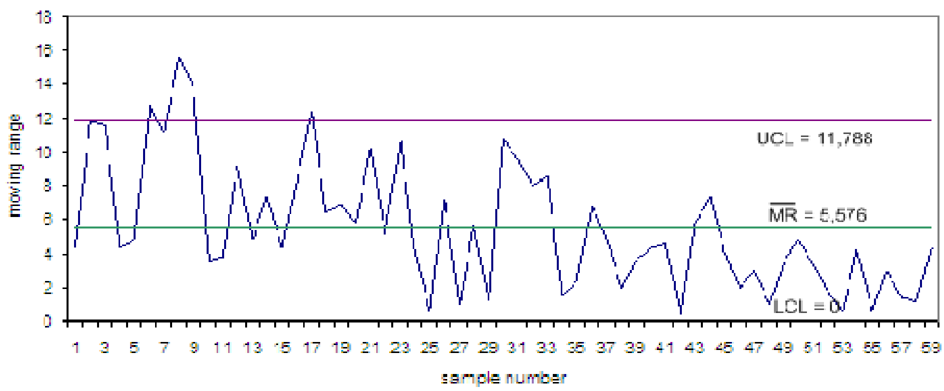
1. Here we give a presentation of the Shewhart control chart for individual measurements known as moving average control chart. Mean charts are not able to detect the difference between routine or natural variations and exceptional variations. However, moving range charts serve to reinforce the mean charts by detecting changes in process variation even if the process average does not change, and providing the ability to discuss the amount of routine variation in a process making the difference between upper and lower natural limits.

Fallowed formulas are used in calculation:

Center line $\overline{MR} = \frac{1}{k-1}\sum_{i=1}^{k} MR_i$

$$UCL_{MR} = \overline{MR} + 3 \cdot d_3 \cdot \sigma_x \quad LCL_{MR} = \max(0, \overline{MR} - 3 \cdot d_3 \cdot \sigma_x)$$

Figure 4. Moving range control chart



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The moving range chart divides variations and presents only the random (routine) variations. Figure 4. shows that the first part of the total observed samples has several points beyond control limits and confirms the above mentioned explanation about the type of pattern samples have. The second part of the time series performs a stationary series contained inside the permitted limits.

2. Exponentially weighted moving average (EWMA) control chart is a statistic for monitoring the process that averages the data that gives less and less weight to data as they are further removed in time. By the choice of weighted factor λ, the EWMA control procedure can be made to a small or gradual shift in a process, whereas Shewhart control procedure can only react when the last data point is outside control limits.

0 < λ ≤ 1 is a constant that determines the depth of memory of the EWMA procedure, or parameter that determines the rate at which older enter into the calculation of the EWMA statistic. Thus, the larger value of λ gives more weight to recent data and less weight to older data, a smaller value of λ gives more weight to older data. Values of λ in the interval 0,05 ≤ λ ≤ 0,25 work well in practice. The use of smaller values of λ is appropriate to detect smaller shifts.

The calculated statistic is:

$$EWMA_i \rightarrow z_i = \lambda \cdot x_i + (1-\lambda) \cdot z_{i-1} \quad \text{where } z_0 = \mu_0$$

$$UCL = \mu + L \cdot \sigma \cdot \sqrt{\frac{\lambda}{(2-\lambda)} \cdot (1-(1-\lambda)^{2i})}$$

$$CL = \mu$$

$$LCL = \mu - L \cdot \sigma \cdot \sqrt{\frac{\lambda}{(2-\lambda)} \cdot (1-(1-\lambda)^{2i})}$$

where L is a number of σ in control limits.

Figure 6. Comparison of raw and exponentially weighted data

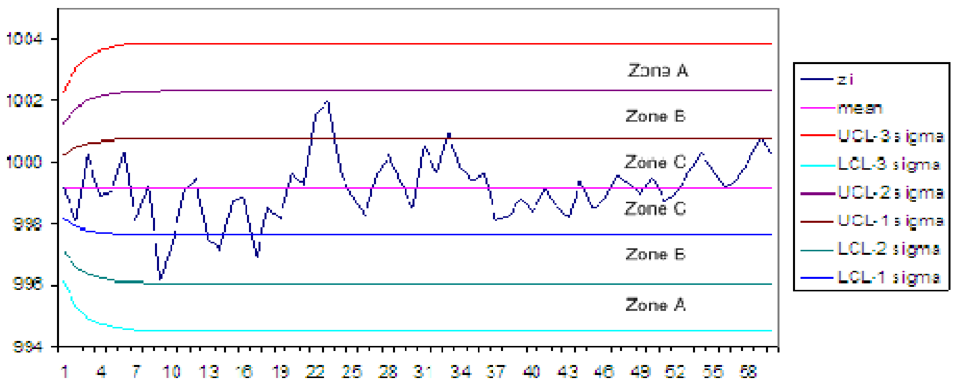

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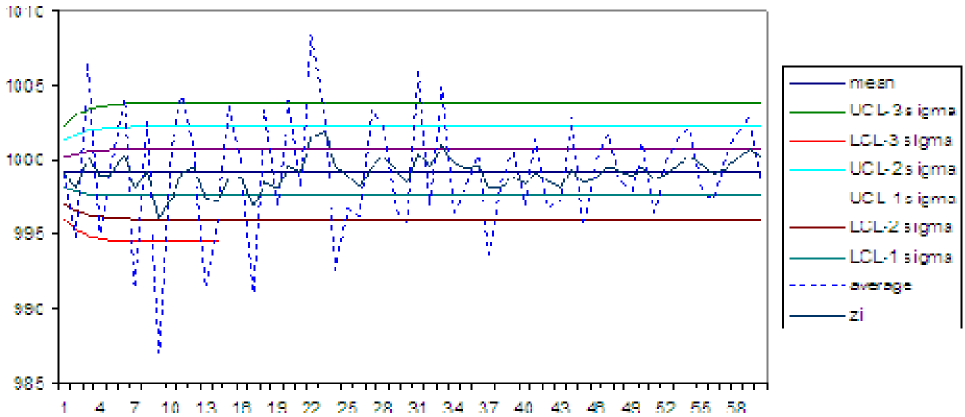
Figure 5. with λ = 0,25 indicates that the process is in control because all EWMA points lie in ± 2σ control limits. Also there is no consecutively big number of sample points in Zone B which indicates the existence of only random or natural variations especially in the first part of the sample. Advanced explanations could use

the comparison between original sample averages and exponentially weighted moving average data presented in Figure 6.

Figure 6. Comparison of raw and exponentially weighted data



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Last figure shows the ability of exponentially weighted data to smooth the affects of known, uncontrollable noise in the data. Many short-term fluctuations may be large, but they are purely indicative of process instability. Appropriate choice of lambda could determine a control chart less sensitive to those short-term fluctuations.

3. Moving average control charts are used to monitor processes over time. MA chart is efficient in detecting small shifts and to evaluate stability of process.

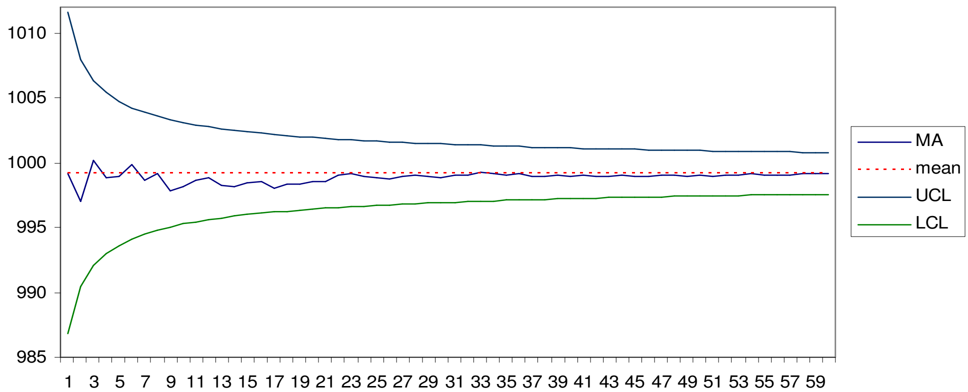
Moving average of span w is $M_i = \frac{x_i + x_{i-1} + \cdots + x_{i-w+1}}{w}$

Where the calculations of the control limits are:

$$UCL = \mu_0 + \frac{3\sigma}{\sqrt{w}} \quad LCL = \mu_0 - \frac{3\sigma}{\sqrt{w}}$$

According to the above formulas control limits are asymptotically approaching to center line simultaneously decreasing the level of permitted variations.

Figure 7. Regression control chart



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This picture clearly presents the lack of variations in the second part of sample group. Moving average line is constantly approaching to the center line and it is showing no sample points out of the retrenched control lines. This is another evidence of improvement in production line process and it could be said that the introductory period for machines or workers is past now and presently the company possesses a completely equipped production line for continuous process improvement.

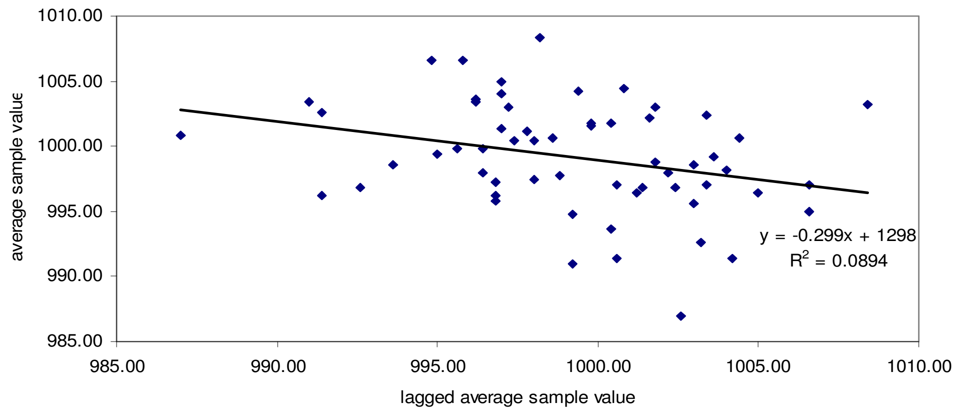
4. The regression control chart is useful when monitoring between two aspects of a production process is important. Using scatter diagram with a regression line could be of great help in identification of outlets in specified regression.

In this paper the following model specification form is used:

$$x_t = \xi + \varphi \cdot x_{t-1} + \varepsilon_t$$

As we can see, the basic autoregressive model is presented here, where ξ and φ are the regression coefficients and ε is residual or model error.

Figure 8. Regression control chart



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Based on simple autoregressive process this figure connects the actual and lagged variable of sample averages. R^2 shows only small percentage of explaining variable variations and this redirect us on the significance of random variations in the explanation of sample values.

5. The modified control charts mentioned above take into account part of the previous data, but the technique which uses all information available is the Cumulative Sum or CUSUM method. This type of charts is one of the most powerful management tools available for the detection of trends and slight changes in data. This chart is useful for detecting a short- and long-term changes and trends. Their interpretation requires care because it is not the actual CUSUM score which signifies the change, but the overall slope of the graph. For this reason the method is often more suitable as a management technique then for use on the operation level.

The method of cumulative differences and plotting them has great application in many fields of management and they provide powerful monitors in such areas as:

- Forecasting
- Absenteeism production level – detection of slight changes
- Maintenance performance
- and many other in which data must be used to signify changes (Oakland 2003)).

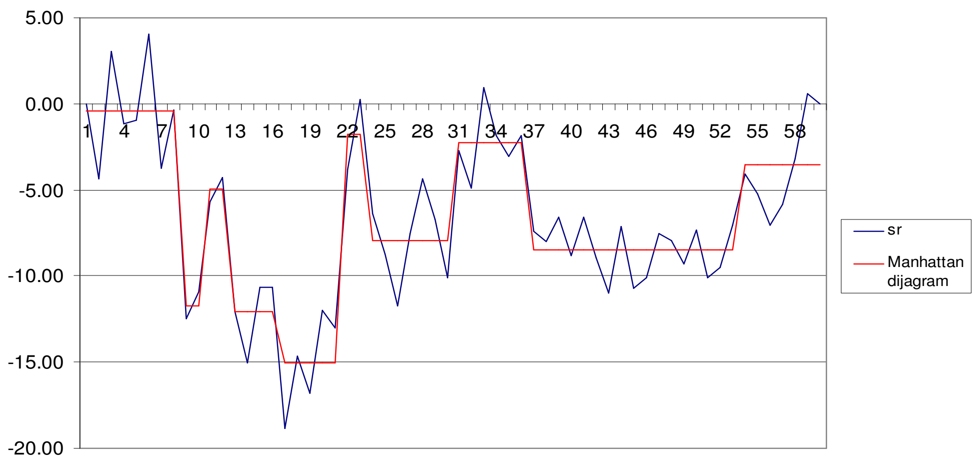
Calculation of the CUSUM score may be represented by the formula:

$$Sr = \sum_{i=1}^{n} (x_i - t)$$

Where
Sr is a the CUSUM score of the *n*th sample
x_i is the result from the individual sample *i* (may be sample mean)
t is called the target value.

The choice of the value of *t* is dependent upon the application of the technique and it is clear that the choice of the *t* value is crucial to the resulting CUSUM graph.

Figure 9. Cumulative sum chart and Manhattan diagram



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This figure present cumulative sum chart with average value as the target value and the Manhattan diagram as the average process mean with time. Because of the constant changes in CUSUM slope, the observations are changing level with many samples below target value. However, second part of the presented observations shows horizontal slope with no significant changes.

Also, CUSUM chart can be used in categorizing process output. This may be for the purposes of selection for different processes or assembly operations, or for dispatch to different customers with slightly varying requirements. To perform the screening or selection, the CUSUM chart is divided into different sections of average process mean by virtue of changes in the slope of the cumulative sum plot. This information may be represented on a Manhattan diagram, named after its appearance. It shows clearly the variations in average process mean over the time scale of the chart.

4.3. Process capability

Another use of control charts not mentioned till now is to determine process capability. Process capability is the range of natural variation in a process known as C_p. It is sometimes also referred to as the natural tolerances of a process. It is used by product designers and engineers to determine how well a process will fall within design specification. In other words, this analysis refers to the infirmity of a process and can be helpful in including development activities prior to manufacturing in analyzing variability relative to product requirements or specifications. Process capability measures potential capability in the process, whereas C_{pk} measures actual capability.

Commonly accepted process capability indices include:

$$C_p = \frac{USL - LSL}{6 \cdot \sigma}$$

$$C_{pu} = \frac{USL - \mu}{3 \cdot \sigma}$$ upper specification limit only

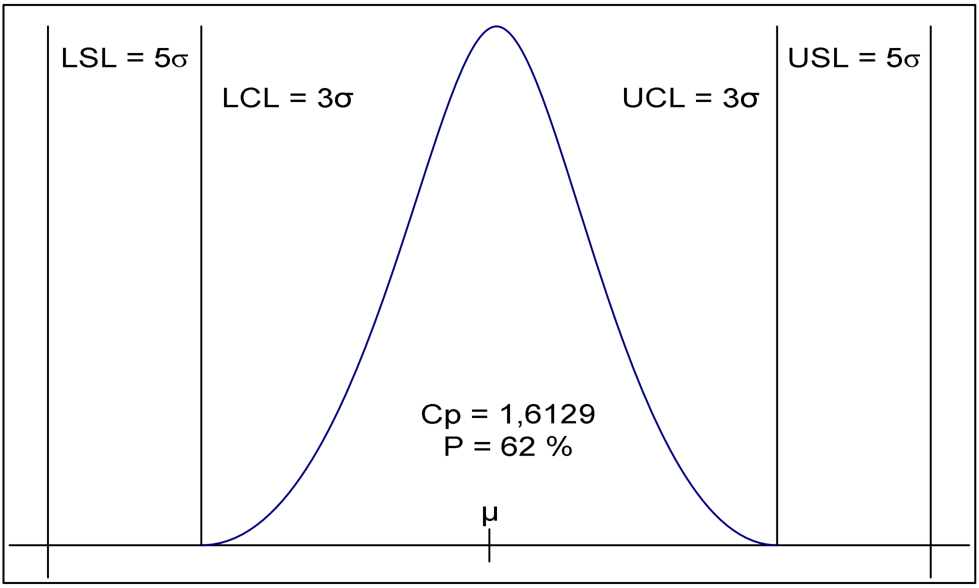
$$C_{pl} = \frac{\mu - LSL}{3 \cdot \sigma}$$ lower specification limit only

$$C_{pk} = \min(C_{pu}, C_{pl})$$ actual process capability

Most capability indices estimates are valid only if the sample number is large enough and assume that the population of data values is normally distributed. Large enough is generally more then 50 independent sample values used in data analyzing.

Specification limits are design or engineering specifications reflecting customer requirements for a product and are set on the level of ± 2 % of the specified product cubage. Process capability is C_p = 1,6129, where upper and lower specification limits are C_{pu} = 1,6782 and C_{pl} = 1,5476. According to this values C_{pk} = 1,5476. Corresponding to previous measures it is possible to estimate percentage of the specification band that the process uses up, $P = \left(\frac{1}{C_p}\right) \cdot 100$. P = 62% of the total allowed natural tolerances.

Figure 10. Process capability and natural tolerances



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Watching this figure we are able to translate process capability into rejects or product failures. Comparison of normal distribution and natural tolerances gives us a probability that any point exceeds the control limits. First, $p_{3\sigma}$ = 0,0027 or 0,27% of failures according to specified control limits, and second, two-sided process capability specification of 5σ is $p_{5\sigma}$ = 0,6 parts per million which is the failure level satisfied the customer specifications.

5. Conclusion remarks

Undoubtedly, efforts undertaken for production process quality improvement will continue. Production line presented in this paper shows that after the methods, machines or workers introduction period had finished, the production has become stabile with constant needs for tracking sudden process changes. Due to this, control charts will require periodic revisions of the control limits and central line in order to make an accurate decision in quality improvement.

As we demonstrated before, the use of the numerous control charts is not only desirable but also necessary due to variety of advantages and disadvantages amongst them.

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